Centre for Excellence in Learning & Teaching

STUDY GUIDE: Mathematical Operations

Observing the special features of the operation EVALUATE

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There exist a variety of mathematical operations some of which change the value of the original while some others change only the appearance of the original. For example, EVALUATE is an operation which always changes the value of the original.

This skill development guide discusses the operation EVALUATE in a question-answer format with relevant examples as outlined below:

Q1 What does the operation EVALUATE involve?

It involves finding the value of a given mathematical expression which has variables and the values suggested for them as shown in the examples below:

Example	**Instruction:** Evaluate the following expressions	**Working & Answer**
01	$4p$ where $p = 3$	$4p = 4(3) = 12$
02	$(a+b)$ where $a = 3, b = 2$	$(a+b) = (3+2) = 5$
03	$(m-n)$ where $m = 3, n = -2$	$m - n = 3 - (-2) = 3 + 2 = 5$
04	$(p+q)$ where $p = -\frac{1}{2}, q = \frac{5}{2}$	$(p+q) = -\frac{1}{2} + \frac{5}{2} = 2$
05	$(y+z)$ where $y = \sqrt{2}, z = \sqrt{3}$	$y + z = \sqrt{2} + \sqrt{3}$

Q2 Would the outcome of EVALUATE operation be always a numerical constant (a number)?

No, not necessarily. Sometimes the outcome can be another variable as illustrated in the examples below:

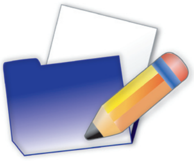
Example	**Instruction:** Evaluate the following expressions	**Working & Answer**
06	$(x+y)$ where $x = a, y = b$	$x + y = a + b$
07	$(m-n)$ where $m = a^2, n = 2c$	$m - n = a^2 - 2c$

Q3 Does EVALUATE operation always involve linear expressions like 2x, m, k + 3, 2w - 5 etc.?

No. Sometimes the EVALUATE operation involves dealing with non-linear expressions which may be quadratic, cubic, exponential, logarithmic, trigonometric, rational or square root types as illustrated in examples below:

Example	**Instruction:** Evaluate the following expressions	**Working & Answer**
08	$(a+b)^2$ where $a = 2, b = y$	$(a+b)^2 = (2+y)^2 = 4 + 4y + y^2$

09	y^2 where $y = s$	$y^2 = s^2$
10	a^x where $a = 2$	$a^x = 2^x$
11	$p + q$ where $p = \log 2,\ q = \log(\frac{1}{2})$	$p + q = \log 2 + \log 2^{-1} = \log 2 + (-1)\log 2 = \log 2 - \log 2 = 0$
12	$(m + n)$ where $m = \frac{1}{x},\ n = \frac{2}{y}$	$m + n = \frac{1}{x} + \frac{2}{y} = \frac{y + 2x}{xy}$
13a	$(a + b)$ when $a = \cos^2\theta, b = \sin^2\theta$	$a + b = \cos^2\theta + \sin^2\theta = 1$
13b	$\sqrt{\frac{p}{q}}$ when $p = 9, q = 25$	$\sqrt{\frac{p}{q}} = \sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$



Q4 Is EVALUATE operation performed only on mathematical expressions?

No. EVALUATE operation is also applied to functions as shown in the following examples:

Example	**Instruction:** Evaluate the following functions	**Working & Answer**
14	$f(x) = 2x$ when $x = -3$	$f(-3) = 2(-3) = -6$ (Ans.)
15	$f(x) = x + \frac{1}{2}$ when $x = \frac{1}{2}$	$f(\frac{1}{2}) = \frac{1}{2} + \frac{1}{2} = 1$ (Ans.)
16	$f(x) = 2x + 5$ when $x = b$	$f(b) = 2b + 5$ (Ans.)
17	$f(x) = 2,$ when $x = -1$	$f(-1) = 2$ (Ans.)
18	$f(x) = 2,$ when $x = 1$	$f(1) = 2$ (Ans.)
19	$f(x) = 2,$ when $x = \frac{1}{2}$	$f(\frac{1}{2}) = 2$ (Ans.)
20	$f(x) = 2,$ when $x = a$	$f(a) = 2$ (Ans.)
21	$f(x) = x^2$ when $x = -2$	$f(-2) = (-2)^2 = 4$ (Ans.)
22	$f(x) = x^2$ when $x = a$	$f(a) = a^2$ (Ans.)
23	$f(x) = x^2 + 5,$ when $x = b$	$f(b) = b^2 + 5$ (Ans.)
24	$f(x) = (x - 3)^2 - 2$ when $x = 1$	$f(1) = (1 - 3)^2 - 2 = (-2)$ (Ans.)
25	$f(x) = x^3$ when $x = -2$	$f(-2) = (-2)^3 = -8$ (Ans.)
26	$f(x) = x^3 - 2x - 2$ when $x = 0$	$f(0) = -2$ (Ans.)
27	$f(x) = x^{-3}$ when $x = -2$	$f(-2) = (-2)^{-3} = \frac{1}{(-2)^3} = -\frac{1}{8}$ (Ans.)
28	$f(x) = \log x,$ when $x = 100$	$f(100) = \log 100 = \log(10)^2 = 2\log 10 = 2(1) = 2$ (Ans.)
29	$f(x) = \sqrt{x + 3}$ when $x = 97$	$f(97) = \sqrt{97 + 3} = \sqrt{100} = 10$ (Ans.)

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30	$f(x) = Sin2x \text{ where } x = \frac{\pi}{3}$	$f(\frac{\pi}{3}) = sin2(60^\circ) = sin\ 120^\circ = \frac{\sqrt{3}}{2}$
31	$f(x) = \frac{1}{x-3}$ when $x = 5$	$f(5) = \frac{1}{5-3} = \frac{1}{2}$ (Ans.)
32	$h(x) = (x - \frac{1}{2})^2$ when $x = m + 2$	$h(m+2) = (m + 2 - \frac{1}{2})^2 =$ $(m + \frac{3}{2})^2 = m^2 + 3m + \frac{9}{4}$ (Ans.)
33	$g(x) = sinx$ when $x = \frac{\pi}{2}$	$g(sinx) = sin\frac{\pi}{2} = 1$ (Ans.)
34	$h(t) = ln(t)^2$ when $t = 10$	$h(t) = ln(t)^2 = 2lnt =$ $2ln(10) = 2$ (Ans.)
35	$f(x+h) - f(x)$ when $f(x) = x^2$, $x = 2$	$f(x+h) - f(x) = (x+h)^2 - x^2$ $= x^2 + 2xh + h^2 - x^2 = 2xh + h^2$ $= 4h + h^2$ (Ans.)
36	$f(g(x))$ when $f(x) = x^2, g(x) = \frac{1}{x^2}$, $x = -2$	$f(g(x)) = f(\frac{1}{x^2}) = (\frac{1}{x^2})^2 = \frac{1}{x^4} = \frac{1}{16}$

Note: Example 35 illustrates a combination of two functions - difference between 2 functions, Example 36 illustrates another type of combination of two functions - composite function.

Q5 Is EVALUATE operation performed only on mathematical expressions and functions?

No. EVALUATE operation is also performed in the context of evaluating limits of functions as shown in the examples below:

Example	**Instruction:** Evaluate the following limits of...
37	$\lim_{x\to 2} \frac{x^2 - 7x + 10}{x^2 - 4} = \lim_{x\to 2} \frac{(x-5)(x-2)}{(x+2)(x-2)} = \lim_{x\to 2} \frac{x-5}{x+2} = \frac{2-5}{2+2} = \frac{-3}{4}$ (Ans.)
38	$\lim_{x\to 25} \frac{5-\sqrt{x}}{25-x} = \lim_{x\to 25} \frac{(5-\sqrt{x})}{(5+\sqrt{x})(5-\sqrt{x})} = \lim_{x\to 25} \frac{1}{5+\sqrt{25}} = \frac{1}{10}$ (Ans.)
39	$\lim_{x\to 9} \frac{9-x}{\sqrt{x}-3} = \lim_{x\to 9} \frac{3^2-(\sqrt{x})^2}{-(3-(\sqrt{x})} = \lim_{x\to 9} \frac{(3+\sqrt{x})(3-\sqrt{x})}{-(3-(\sqrt{x})} =$ $-(3+\sqrt{9}) = -6$ (Ans.)
40	$\lim_{x\to 0} \frac{(x+3)^3 - 27}{x} = \lim_{x\to 0} \frac{(x+3-3)[(x+3)^2 + 3(x+3) + 9]}{x}$ $\lim_{x\to 0} \frac{x(x^2 + 6x + 9 + 3x + 9 + 9)}{x} = \lim_{x\to 0} (x^2 + 9x + 27) = 27$ (Ans.)
41	$\lim_{x\to 0} \frac{3}{x}\left(\frac{1}{5+x} - \frac{1}{5-x}\right) = \lim_{x\to 0} \frac{3}{x}\left(\frac{5-x-5-x}{(5+x)(5-x)}\right) = \lim_{x\to 0} \frac{3}{x}\left(\frac{-2x}{25-x^2}\right) =$ $\lim_{x\to 0} \frac{3}{x}\left(\frac{-6}{25-x^2}\right) = \frac{-6}{25}$ (Ans.)
42	$\lim_{x\to 4} \frac{(x-4)^3}{\|4-x\|} = \lim_{x\to 4} \frac{(x-4)\|x-4\|^2}{\|-(x-4)\|} = \lim_{x\to 4} \frac{(x-4)\|x-4\|^2}{\|x-4\|} =$ $\lim_{x\to 4} (x-4)\|x-4\| = 0$ (Ans.)

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43	$\lim\limits_{x \to 0} \frac{x\mathrm{Sin}(x)}{\lvert x \rvert}$ Since $-1 \le \frac{x}{\lvert x \rvert} \le 1$, we have $-\sin(x) \le \frac{x\mathrm{Sin}(x)}{\lvert x \rvert} \le \sin(x)$ By Squeeze theorem, $\lim\limits_{x \to 0} \frac{x\mathrm{Sin}(x)}{\lvert x \rvert} = 0$ (Ans.)

Q6 Which other mathematical contexts use the operation EVALUATE?

EVALUATE operation is also performed in the context of formal definition (ε , δ) of limits of functions as shown in the examples below:

Example	**Instruction:** Evaluate δ when:
44	$\lim\limits_{x \to 2} 7x + 4 = 18$ and $\varepsilon = .01$ We need to calculate: $\lvert 7x + 4 - 18 \rvert < \varepsilon \Rightarrow \lvert 7x - 14 \rvert < .01 \Rightarrow 7\lvert x - 2 \rvert < .01$ $\Rightarrow \lvert x - 2 \rvert < \frac{.01}{7}$ Hence $\boldsymbol{\delta = \frac{.01}{7} = \frac{1}{700}}$
45	$\lim\limits_{x \to 3} x^2 = 9$ and $\varepsilon = .05$ We need to calculate: $\lvert x^2 - 9 \rvert < .05 \Rightarrow -.05 < x^2 - 9 < .05$ $\Rightarrow -.05 + 9 < x^2 < 9 + .05 \Rightarrow 8.95 < x^2 < 9.05$ $\Rightarrow 2.9916... < x < 3.0083... \Rightarrow 2.9916... - 3 < x - 3 < 3.0083... - 3$ $\Rightarrow -.0083... < x - 3 < .0083...$ Hence $\boldsymbol{\delta = .0083}$
46	$\lim\limits_{x \to 24} \sqrt{x+1} = 5$ and $\varepsilon = .1$ We calculate: $\left\lvert \sqrt{x+1} - 5 \right\rvert < .1 \Rightarrow -.1 < \sqrt{x+1} - 5 < .1 \Rightarrow 5 - .1 <$ $\sqrt{x+1} < 5 + .1 \Rightarrow 4.9 < \sqrt{x+1} < 5.1 \Rightarrow 24.01 < x + 1 < 26.01 \Rightarrow$ $23.01 < x < 25.01 \Rightarrow 23.01 - 24 < x - 24 < 25.01 - 24 \Rightarrow -.99 <$ $x - 24 < 1.01$ Hence $\boldsymbol{\delta = .99}$ (δ is to be taken as the smaller of the two values)
47	$\lim\limits_{x \to \frac{\pi}{4}} \mathrm{Sin}\, x = \frac{\sqrt{2}}{2}$ and $\varepsilon = .003$ We calculate: $\left\lvert \sin x - \frac{\sqrt{2}}{2} \right\rvert < .003 \Rightarrow -.003 < \mathrm{Sin}\, x - \frac{\sqrt{2}}{2} < .003$ $\Rightarrow \frac{\sqrt{2}}{2} - .003 < \mathrm{Sin}\, x < \frac{\sqrt{2}}{2} + .003 \Rightarrow \mathrm{Sin}^{-1}\left(\frac{\sqrt{2}}{2} - .003\right) < x < \mathrm{Sin}^{-1}$ $\left(\frac{\sqrt{2}}{2} + .003\right) \Rightarrow .78116... - \frac{\pi}{4} < x - \frac{\pi}{4} < .78964... - \frac{\pi}{4} \Rightarrow -.00423 <$ $x - \frac{\pi}{4} < .00425$ Hence $\boldsymbol{\delta = .00423}$
48	$\lim\limits_{x \to 1} \frac{1}{x} = 1$ and $\varepsilon = .07$ $\left\lvert \frac{1}{x} - 1 \right\rvert < .07 \Rightarrow -.07 < \frac{1}{x} - 1 < .07 \Rightarrow .93 < \frac{1}{x} < 1.07 \Rightarrow \frac{1}{1.07} < x$ $< \frac{1}{.93} \Rightarrow .93457... < x < 1.0752... \Rightarrow .93457... - 1 < x - 1 < 1.0752... - 1$ $\Rightarrow .06543 < x - 1 < .0752$ Hence $\boldsymbol{\delta = .06543}$

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